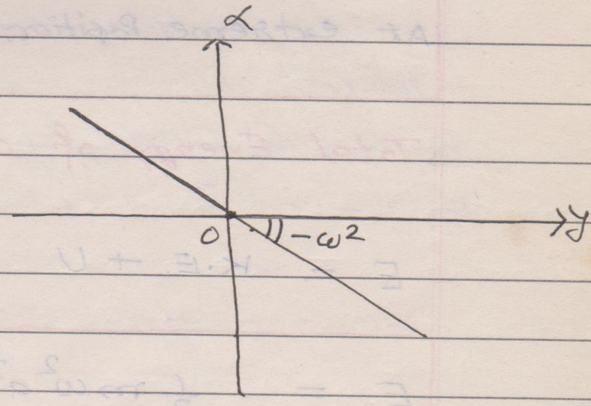
(iv) The velocity-displacement graph for simple harmonic motion $y = a\sin\omega t$ and $v = a\omega\cos\omega t$ is ellipse. if $\omega = 1$ then it is represented by a circle.

$$\frac{y}{a} = \sin\omega t \quad \& \quad \frac{v}{a\omega} = \cos\omega t$$

$$\boxed{\frac{y^2}{a^2} + \frac{v^2}{a^2\omega^2} = 1 \longrightarrow \text{Ellipse}}$$

If $\omega = 1$ then $\boxed{y^2 + v^2 = a^2 \longrightarrow \text{circle.}}$

(v) The acceleration-displacement graph is in simple harmonic motion $y = a\sin\omega t$ and $\alpha = -\omega^2 y = -\omega^2 a\sin\omega t$ is a straight line whose slope is $-\omega^2$



(vi) The velocity-acceleration graph in S.H.M. where $v = a\omega\cos\omega t$ and $\alpha = -\omega^2 a.\sin\omega t$ is an ellipse because.
$\frac{v^2}{a^2\omega^2} + \frac{\alpha^2}{a^2\omega^4} = 1$. which is an equation of ellipse.

Kinetic Energy of a Particle executing S.H.M. :-

$$\boxed{K.E. = \frac{1}{2} m v^2 = \frac{1}{2} m\omega^2 (a^2 - y^2)}$$

$K.E. = \frac{1}{2} m\omega^2 a^2 \cancel{\cos}^2 \omega t$

$= \frac{1}{4} m\omega^2 a^2 (1 + \cos 2\omega t)$

At mean Position $y = 0$ $\therefore K.E. = \frac{1}{2} m\omega^2 a^2 = \text{Maximum}$

At extreme Position $y = a$, $K.E. = 0 = \text{minimum}$

Potential Energy of a Particle executing S.H.M.:-

$$\boxed{U = \frac{1}{2} m\omega^2 y^2} = \frac{1}{2} m\omega^2 a^2 \sin^2 \omega t = \frac{1}{4} m\omega^2 a^2 (1 - \cos 2\omega t)$$

At mean Position $y = 0$, $U = 0 = \text{minimum}$

At extreme Position $y = a$, $U = \frac{1}{2} m\omega^2 a^2 = \text{maximum}$

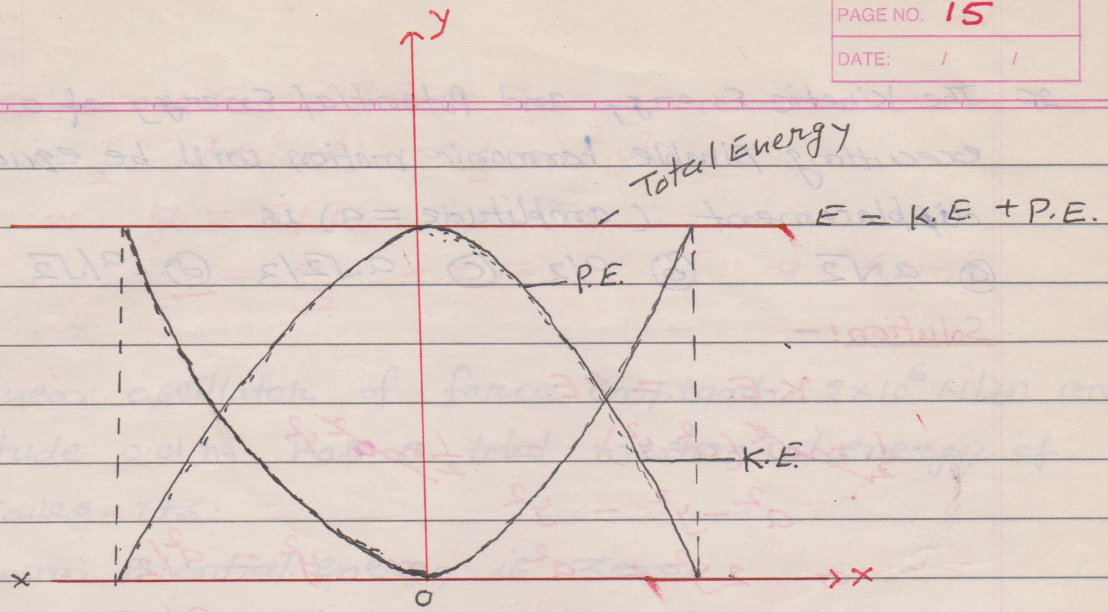
Total Energy of a Particle executing S.H.M. :-

$$E = K.E. + U = \frac{1}{2} m\omega^2 (a^2 - y^2) + \frac{1}{2} m\omega^2 y^2$$

$$\boxed{E = \frac{1}{2} m\omega^2 a^2} \boxed{= 2\pi^2 m a^2 f^2}$$

where f is the frequency.

* Total Energy of a Particle executing S.H.M. remains Constant during the motion.
* If the frequency of S.H.M. is f ~~[illegible]~~ and time period T then the frequency of K.E. & P.E. is $2f$ and Time Period of K.E. & P.E. is $T/2$.


y
Total Energy
E = K.E + P.E.
P.E.
K.E.
x
o
x

26. The Kinetic Energy and Potential Energy of a Particle executing simple harmonic motion will be equal, ~~t~~when displacement (amplitude $= a$) is

(a) $a\sqrt{2}$ (b) $a/2$ (c) $a\sqrt{2}/3$ (d) $a/\sqrt{2}$

Solution:-

$$K.E. = P.E.$$

$$\frac{1}{2}\cancel{m\omega^2}(a^2 - y^2) = \frac{1}{2}\cancel{m\omega^2}y^2$$

$$a^2 - y^2 = y^2$$

or $2y^2 = a^2$ or $y^2 = a^2/2$

or $y = a/\sqrt{2}$

27. A Point Particle of mass 0.1 Kg is executing S.H.M. of amplitude 0.1 m. when the Particle passes through the mean Position its K.E. is 8×10^{-3} J. The equation of motion of this particle, if its initial Phase of oscillation is 45° is

(a) $y = 0.1 \sin\left(\frac{t}{4} + \frac{\pi}{4}\right)$ (b) $y = 0.1 \sin(t/2 + \pi/4)$

(c) $y = 0.1 \sin(4t - \pi/4)$ (d) $y = 0.1 \sin(4t + \pi/4)$

Solution:- amplitude $a = 0.1$, $\phi = 45^\circ = \pi/4$

$$K.E. = \frac{1}{2}m\omega^2a^2 = 8 \times 10^{-3}$$

$$\frac{1}{2} \times 0.1 \times \omega^2 \times (.1)^2 = 8 \times 10^{-3}$$

$$\omega^2 = 16$$

$$\omega = 4$$

$$\therefore y = a \sin(\omega t + \phi)$$

$$y = 0.1 \sin(4t + \pi/4)$$

28. A Particle is vibrating in a simple harmonic motion with an amplitude of 4 cm. At what displacement from the equilibrium position, its Energy half Potential and half Kinetic?

(a) 3 cm (b) $2\sqrt{2}$ cm (c) 1 cm (d) $\sqrt{2}$ cm.

Solution:- K.E. = P.E.

$$\frac{1}{2}m\omega^2(a^2-y^2) = \frac{1}{2}m\omega^2y^2$$

or $y = a/\sqrt{2}$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2}\text{ cm} \quad [\because a = 4\text{ cm}]$$

29. A linear oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 Joules. Its:

(a) minimum Potential energy is zero
(b) maximum Potential energy is 160 J.
(c) maximum Kinetic energy is 100 J.
(d) maximum Potential energy is 100 J.

Solution: Total mechanical energy = maximum Potential energy = 160 J.

Maximum Kinetic energy $= \frac{1}{2}Ka^2 = \frac{1}{2}(2\times10^6)\times(0.01)^2 = 100$ J

30. When the displacement is half of the amplitude, then what fraction of the total energy of a simple harmonic oscillator is kinetic?

(a) $\frac{2}{7}$ th (b) $\frac{3}{4}$ th (c) $\frac{2}{9}$ th (d) $\frac{5}{7}$ th.

Solution:- $K.E. = \frac{1}{2}m\omega^2(a^2-y^2)$

$K.E. = \frac{1}{2}m\omega^2\left(a^2-\frac{a^2}{4}\right)$ $\quad \because y = a/2$

$$K.E. = \frac{1}{2}m\omega^2\cdot\frac{3}{4}a^2 = \frac{3}{4}\left(\frac{1}{2}m\omega^2a^2\right)$$

$K.E. = \frac{3}{4}(\text{total Energy})$ $\quad \because$ Total Energy $= \frac{1}{2}m\omega^2a^2$

31. The kinetic energy of a particle executing S.H.M. is 16 J when it is in its mean position. If the amplitude of oscillation is 25 cm and the mass of the particle is 5.12 kg, the time period of its oscillation is:

(a) 2π sec (b) 20π sec (c) 5π sec (d) $\frac{\pi}{5}$ sec

Solution: - K.E. $= \frac{1}{2} m\omega^2 a^2 = 16$

or $\frac{1}{2} \times 5.12 \times \omega^2 \times (25 \times 10^{-2})^2 = 16$

$$\omega^2 = \frac{32}{5.12 \times 625 \times 10^{-4}} = \frac{\cancel{32}^{4} \times 10^4 \times \cancel{100}^{4}}{\cancel{625}_{25} \times \cancel{512}_{\cancel{64}\,16}} = 100$$

or $\omega = 10$ rd/sec.

$T = 2\pi/\omega = \frac{2\pi}{10} = \frac{\pi}{5}$ sec

32. When the Potential energy of a Particle executing S.H.M. is one fourth of its maximum value during the oscillation, the displacement of the Particle from the equilibrium Position in terms of its amplitude 'a' is —

(a) $a/3$ (b) $a/2$ (c) $2a/3$ (d) $a/4$.

Solution: P.E. $= \frac{1}{4} \times$ Maximum Value of P.E.

$\frac{1}{2} \cancel{m\omega^2} y^2 = \frac{1}{4} \times \frac{1}{2} \cancel{m\omega^2} a^2$

$y^2 = a^2/4$ or $y = a/2$

33. A Particle of mass 10 gm is describing S.H.M. along a straight line with Period 2 sec. and amplitude of 10 cm. Its Kinetic energy when it is at 5 cm from its equilibrium Position is: -

(a) $3.75\pi^2$ ergs (b) $375\pi^2$ ergs

(c) $0.375\pi^2$ ergs (d) $37.5\pi^2$ ergs.

Solution: K.E. $= \frac{1}{2} m\omega^2 (a^2 - y^2)$

$= \frac{1}{2} m \frac{4\pi^2}{T^2}(a^2 - y^2) = \frac{2m\pi^2(a^2-y^2)}{T^2}$

$= \frac{2 \times 10^{5} \times \pi^2 (100 - 25)}{4 \cancel{4}} = 5\pi^2 \times 75$

$= 375\pi^2$ ergs

34. In simple harmonic motion displacement is half of its amplitude, what will be the ratio of K.E. & P.E.

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $4/3$ (d) None.

Solution:-
$$\frac{K.E.}{P.E.} = \cancel{\tfrac{1}{2} m\omega^2}(a^2-y^2) / \cancel{\tfrac{1}{2} m\omega^2} y^2$$
$$= \frac{a^2-y^2}{y^2} = \frac{a^2-a^2/4}{a^2/4} \quad [\because y = a/2$$
$$= \frac{3a^2/4}{a^2/4} = 3$$

35. In simple pendulum, if K.E. is one fourth of total energy, then displacement (x) is related to amplitude A, as—

(a) $x = A$ (b) $x = 2A$ (c) $x = \frac{\sqrt{3}}{4}A$ (d) $x = \frac{\sqrt{3}}{2}A$.

Solution:- K.E. $= \frac{1}{4}$ Total Energy
$$\cancel{\tfrac{1}{2} m\omega^2}(A^2-x^2) = \frac{1}{4} \times \cancel{\tfrac{1}{2} m\omega^2} A^2$$
$$A^2 - x^2 = \frac{A^2}{4} \text{ or } x^2 = A^2 - \frac{A^2}{4} = \frac{3A^2}{4}.$$
$$\text{or } x = \frac{\sqrt{3}}{2}A.$$

—×—

Simple Harmonic oscillations of springs:-

Spring have a property that when they are stretched or compressed then a restoring force immediately developed in them which try to bring them back to their initial state. If the extension produced in the spring is 'x' then

$$F = -Kx$$

where K is a constant known as spring constant or force constant. K is numerically equal to

$$K = \frac{F}{x} = mg/x$$

Its unit is Newton/meter & dimentional formula is $[MT^{-2}]$

The time period of a spring loaded by mass 'm' is

$$\boxed{T = 2\pi\sqrt{m/K} = 2\pi\sqrt{x/g}}$$

* If the spring of spring constant K, cut into n equal parts, then the spring constant of each part will become Kn. If the spring is stretched such that its length becomes n times the previous length then the spring constant of the spring becomes K/n

* The spring constant of the combination of two springs in series is

$$\boxed{\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} \quad \text{or} \quad K = \frac{K_1K_2}{K_1+K_2}}$$

* The spring constant of the combination of two springs in Parallel is

$$\boxed{K = K_1 + K_2}$$

* If a body of mass 'm' is suspended from one end of a spring whose mass is m_s and spring constant 'K' time Period of vibration is given by:

$$\boxed{T = 2\pi\sqrt{\frac{m + m_s/3}{K}}}$$

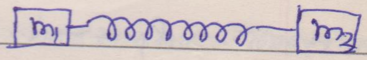
* If n Identical springs are connected in series each of spring constant K, then effective spring constant $K' = K/n$ and effective time Period

$$\boxed{T' = \sqrt{n}\, T}$$

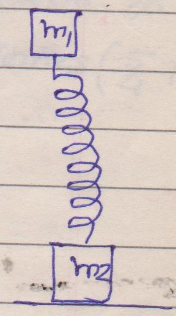
* If n Identical spring are connected in Parallel each of spring constant K, then effective spring constant $K' = K \cdot n$ and effective time Period

$$\boxed{T' = T/\sqrt{n}.}$$

* In a two Particle system as shown in fig., if K is the force constant of the spring, then time Period of vibration of system is given by



$$\boxed{T = 2\pi\sqrt{\frac{\mu}{K}}}$$



$$\boxed{\text{where } \mu = \frac{m_1 m_2}{m_1 + m_2} = \text{reduce mass}}$$

* In a system of two Particle if only one mass m_1 is oscillating then time period

$$\boxed{T = 2\pi\sqrt{\frac{m_1}{K}}} \quad \boxed{\& \ K = (m_1+m_2)g/l}$$

* If a body connected to a spring rolls, then the time period of its S.H.M. is given by

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}\left(1+\frac{k^2}{r^2}\right)}}$$

where k is the radius of gyration of the body about an axis passing through it C.G.

* If a mass is suspended from a spring and mad to oscillate in a liquid, its time period remains unchan

* When a spring of spring constant K and of length l is cut into two pieces of length l_1 and l_2 such that $l_1 = nl_2$, where n is whole number, the spring constan of length l_1 is $\boxed{K_1 = \frac{K(n+1)}{n}}$

and of length l_2 is $\boxed{K_2 = (n+1)K}$

$$l_1 + l_2 = l \qquad \frac{K}{K_2} = \frac{l_2}{l}$$

$$nl_2 + l_2 = l$$

$$l_2 = \frac{l}{n+1} \qquad K_2 = \frac{Kl}{l_2} = \frac{Kl(n+1)}{l}$$

$$l_1 = \frac{nl}{n+1} \qquad K_1 = \frac{Kl}{l_1} = \frac{Kl(n+1)}{nl}$$

36. A mass m is suspended from the two coupled springs connected in series. The force constant for springs are K_1 and K_2. The time period of the suspended mass will be:

(a) $T = 2\pi\sqrt{\frac{m}{K_1 - K_2}}$ (b) $T = 2\pi\sqrt{\frac{m K_1 K_2}{K_1 + K_2}}$

(c) $T = 2\pi\sqrt{\frac{m}{K_1 + K_2}}$ (d) $T = 2\pi\sqrt{\frac{m(K_1+K_2)}{K_1 K_2}}$

37. A spring having a spring constant 'K' is loaded with a mass 'm'. The spring is cut into two equal parts and one of these is loaded again with the same mass. The new spring constant is:—

(a) K (b) 2K (c) K/2 (d) K^2

38. A spring has a force constant K and a mass 'm' is suspended from it. The spring is cut in half and the same mass is suspended from one of the halves. If the frequency of oscillation in the first case is n, then frequency in the second case will be:

(a) n (b) n/2 (c) $n\sqrt{2}$ (d) 2n.

39. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes simple harmonic ~~motion~~ oscillation with a time period T. If the mass is increased by m, then the time period becomes $\left(\frac{5}{4}T\right)$. The ratio of $\frac{m}{M}$ is

(a) $\frac{9}{16}$ (b) $\frac{5}{4}$ (c) $\frac{25}{16}$ (d) $\frac{4}{5}$

Solution:— $T = 2\pi\sqrt{\frac{M}{K}}$, $T' = 2\pi\sqrt{\frac{M+m}{K}}$

$$\frac{T'}{T} = \sqrt{\frac{M+m}{M}} = \sqrt{1 + \frac{m}{M}}$$

$$\left(\frac{5}{4}\right)^2 = 1 + \frac{m}{M} \quad \text{or} \quad \frac{m}{M} = \frac{9}{16}$$

40. The length of the spring is l and its force constant is K. When a weight 'W' is suspended from it, its length increases by x. If the spring is cut into two equal parts and put in Parallel the same weight 'W' is suspended from them, then the extension will be

ⓐ x ⓑ $2x$ ⓒ $x/4$ ⓓ $x/2$;

Solution:- $W = Kx$. If K is the force constant of each two equal parts of spring, then $K = 2K$. when two parts of spring are connected in Parallel to a load the effective spring constant $K' = 2K + 2K = 4K$

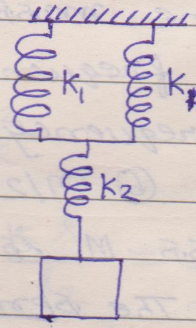
$$\therefore W = K'x_1 = 4Kx_1$$

$$\text{or } x_1 = \frac{W}{4K} = \frac{Kx}{4K} = x/4$$

41. What ~~is the~~ will be the force constant of the spring system shown in fig

ⓐ $\frac{K_1}{2} + K_2$ ⓑ $\left[\frac{1}{2K_1} + \frac{1}{K_2}\right]^{-1}$

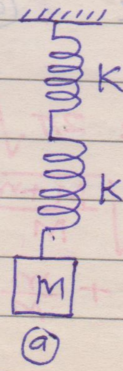
ⓒ $\frac{1}{2K_1} + \frac{1}{K_2}$ ⓓ $\left[\frac{2}{K_1} + \frac{1}{K_2}\right]^{-1}$



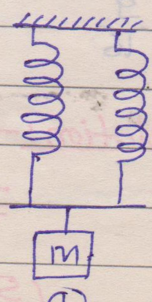
42. Two Identical springs of constant K are connected in series and Parallel as shown in fig. A mass M is suspended from them. The Ratio of their frequencies of vertical oscillations will be:

ⓐ 2 : 1 ⓑ 1 : 2

ⓒ 1 : 4 ⓓ 4 : 1



ⓐ



ⓑ